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PHENOMENOLOGICAL THEORY OF ENERGY AND MASS TRANSFER FOR PRESSURE SHAPING OF DISPERSE MATERIALS

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Abstract—In the paper a physico-mathematical model of moulding is proposed that reduces a nonlinear problem of energy and mass transfer with volume deformation to a linear problem of energy and mass transfer to a thermodynamic system with a constant volume.

NOMENCLATURE

μ, mass-transfer potential [J/kg^3];
T, absolute temperature;
x, y, z, Cartesian coordinates [m];
s, entropy of unit mass [J/kg°K];
s_v, entropy concentration [J/m^3°K];
v, $= 1/\rho$, specific volume [m^3/kg];
$\rho, \rho_m, \Delta\rho_m$, density, mean density, increment of mean density, respectively [kg/m^3];
P, P_{ij}, pressure, normal and tangential stresses $(i, j = 1, 2, 3)$, respectively;
P_x, P_y, P_z, physical scalars of normal stresses along axes x, y, z, respectively;
P_s, P_c, pressure over the surface and at the centre of the disperse plate;
P_{so}, P_{co}, P_{se}, pressure over the surface and at the centre of disperse plate at the beginning and end of the time interval, respectively;
P_{si}, P_o, pressure at the ith period of moulding and pressure increment at the beginning of region $\Delta\tau$;
P_m^b, P_m^e, mean pressure in the beginning and at the end of time interval ("zone") [N/m^2];
e, concentration of compression energy per per unit volume [J/m^3];
F, energy and mass-receiving surface [m^2];
$C_e, (C_e)_m, (C_e)_O^{P_m}, (C_e)_{P_m^b}^{P_m^e}$, energy capacity coefficient and its mean values, respectively: mean, mean within pressure range between O and P_m; mean within pressure range between P_m^b and P_m^e [J/m^3(N/m^2)];
$C_{em}, (C_{em})_O^{\Delta\rho_m}, (C_{em})_{\Delta\rho_b}^{\Delta\rho_e}$, specific energy coefficient and its mean values, respectively: within density increment range between O and $\Delta\rho_m$, and between $\Delta\rho_b$ and $\Delta\rho_e$
$$\left[\frac{\text{J}}{\text{m}^3(\text{kg/m}^3)}\right];$$
q_m, specific mass flow [kg/m^2 s];
q_e, q_{eo}, q_{em}, specific flux of mechanical energy and its values, respectively at the initial moment of interval $\Delta\tau$("O") and mean value within this interval (m)
$$\left[\frac{\text{W}}{\text{m}^2}\right];$$
ξ_p, rate of changing pressure $\left[\frac{\text{N/m}^2}{\text{s}}\right]$;
ξ_q, rate of changing specific energy flux
$$\left[\frac{\text{W}}{\text{m}^2\text{s}}\right];$$
R, predicted thickness of disperse plate [m];
Fo, Fourier number;
θ, entropy production per unit volume
$$\left[\frac{\text{J}}{\text{m}^3\text{s}}\right];$$
X_m, thermodynamic motive force of mass transfer;
a_e, a_{em}, potential conductivity and its mean value in predicted "zone" [m^2/s];
λ, energy conductivity [W/m^2 (N/m^2)];
M_w, M_{in}, M_{idl}, working power, intake power and idling power of power unit, respectively [W].

INTRODUCTION

RECENT WORKS of Dutch and Belgian physicists, mainly of de Groot, and of Soviet scientists under general direction of Academician A. V. Luikov have allowed a new powerful method to be developed for phenomenological investigation of transfer processes called thermodynamics of irreversible processes.

The presented paper is the first attempt to treat pressure shaping of disperse materials from the phenomenological point of view.

THE MAIN POINTS OF THE FORMULATED MODEL

A disperse material in a volume restricted by the required dimensions of a manufactured product and the press mould walls is assumed as a thermodynamic system. The disperse material is a continuous system with parameters such as density, pressure, temperature always variable in space when interacting with the surrounding medium.

The objective of moulding is to increase mass content of the thermodynamic system by addition of mass (homogeneous or inhomogeneous) from the surrounding medium.

"Excess" mass of the material under plungers, the plungers themselves with a drive are the surrounding medium, a kind of "mass-energostat", for the present system to exchange mass and energy with. The "mass-energostat" is assumed to possess a sufficient amount of mass and energy for the present technological process to run. During the whole moulding cycle mass and energy concentrations in it are not a function of the spatial coordinates.

External mass and energy transfer between the thermodynamic system surface and the surrounding medium occurs due to replacement of an elementary surface layer of the system with a small energy potential by the mass from the "mass-energostat" with a higher margin of the same potential in the direction of the external force.

Inside the system considered processes of successive movement of elementary layers of mass towards the system axis, transfer of momentum, kinetic energy at collisions of discrete mass particles proceed in a random diffusive way, involve energy dissipation for internal friction forces and are damping. This is the way how diffusive mass and energy fluxes arise. Thus, a complex process of mass packing with deformation of the disperse material volume in the press mould is represented by a simplified physical model of mass and energy transfer in a thermodynamic system with a constant volume.

A disperse material is placed within a field of conservative earth gravitation forces. It is shown that a natural process of consolidation (packing the material due to these body forces) proceeds in relatively thin plates for years. Therefore, the gravity effect on distribution of the disperse material density may be neglected, i.e.

$$\rho_o \neq f(Z).$$

A uniform initial distribution of the material over two other axes in the plate is also assumed

$$\rho_o \neq f(X, Y, Z, O).$$

A MATHEMATICAL MODEL OF THE PROCESS

A moulding process will now be treated that proceeds at bilateral symmetrical mass-energy flux supply to a disperse plate of an infinite length and width (Fig. 1).

Mass and energy transfer into the system is accompanied by disturbance of its mass-energy equilibrium with the surrounding medium. All changes that take place in the system during an elementary irreversible process are included in the second principle of thermodynamics by the entropy production term which at initial assumptions of the process model is of the form

$$T\,dS_v = \mu\,d\rho. \tag{1}$$

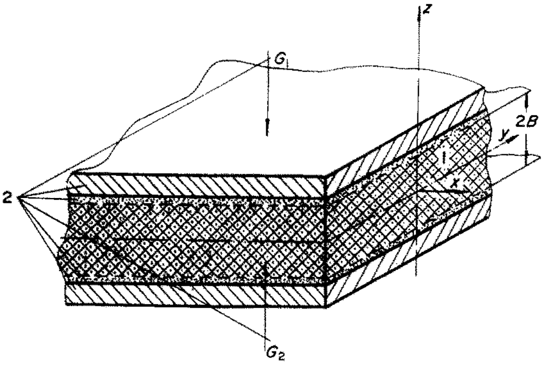

FIG. 1. Geometric interpretation of the physical model of mass and energy transfer for pressure shaping of an infinite disperse plate. $G_1 = G_2$ are forces applied to plungers (2) at symmetric bilateral moulding; 1, thermodynamic system with prescribed volume $V = F.2B$ being saturated with mass and energy; F, mass- and energy-receiving surface area of the system, m^2. $2B$, disperse plate thickness, m; 2, "mass-energostat", a source of mass and energy at moulding (plungers and excessive mass saturating the thermodynamic system); x, y, z, Cartesian coordinates.

The Gibbs–Duhem equation

$$d\mu = s\,dT + v\,d\rho, \quad \text{J/kg}$$

gives the mass-transfer potential μ, where pressure, density, temperature, concentration and the other scalar quantities are physical scalars. In this case, for liquids in equilibrium with the surrounding medium, pressure P represents compression energy storage per unit volume, N/m^2, N.m/m^3, J/m^3, i.e. $P = e$.

A stressed state of disperse materials is characterized by the antisymmetric tensor of the second order

$$P^a = \begin{Bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{Bmatrix} \tag{3}$$

where $P_{11} = -P_x.\mathbf{n}$; $P_{22} = P_y.\mathbf{n}$; $P_{33} = P_z.\mathbf{n}$.

The total compression energy storage in the system under stress is not included by any of the scalar quantities P_i $(i = X, Y, Z)$.

Therefore, the specific energy consumed by the mass per unit volume of this system may be expressed as

$$e = P_z.C_e \tag{4}$$

$$\text{where } C_e = \frac{\partial e}{\partial P_z}; \quad \text{for liquids } \frac{\partial e}{\partial P_z} = 1; \quad e = P.$$

So, at $dT \approx 0$ in accordance with equation (2) we have

$$d\mu = v\,de \tag{5}$$

hence

$$\mu = ve \tag{6}$$

$$T\,ds_v = ve\,d\rho \tag{7}$$

or for local interpretation

$$\frac{\partial s_v}{\partial \tau} = -\frac{v}{T}e\frac{\partial \rho}{\partial \tau}, \quad \text{J/m}^3\text{.deg.s.} \tag{8}$$

With account for Umov equation [1] after some transformations we arrive at

$$\frac{\partial s_v}{\partial \tau} = \frac{1}{T}\operatorname{div}(veq_m) - q_m \frac{v}{T}\operatorname{grad} e. \tag{9}$$

The energy dissipation within the considered irreversible process is described by the second term in the l.h.s. of equation (9)

$$\left(T\frac{\partial s_v}{\partial \tau}\right)_{ir} = T\theta = -q_m . v\frac{\partial e}{\partial Z} \tag{10}$$

where $\theta = (\partial s_v/\partial \tau)_{ir}$ is the rate of local entropy production in the system due to energy dissipation in irreversible processes.

Hence, following Onsager [2], thermodynamic motive force of mass transfer X_m at moulding may be found from the expression

$$-q_m v\frac{\partial e}{\partial Z} = q_m X_m \tag{11}$$

as

$$X_m = -v\frac{\partial e}{\partial Z} \tag{12}$$

or

$$X_m = -v\frac{\partial (P_z . C_e)}{\partial Z}. \tag{13}$$

The Onsager linear relation for a flow and thermodynamic force requires the equality to hold

$$q_m = L_m . X_m = -L_m . v\frac{\partial e}{\partial Z} \tag{14}$$

where L_m is the kinetic coefficient.

The dimensional analysis shows that $L_m . v$ may be expressed as

$$L_m . v = \frac{a_e}{C_{em}}. \tag{15}$$

So, a specific mass flux to the system with a finite thickness equals

$$q_m = -\frac{a_e}{C_{em}} . \frac{\partial (P_z . C_e)}{\partial Z}, \quad \text{kg/m}^2\,\text{s}. \tag{16}$$

From the Umov equation and equation (16) for zonal calculation (at C_e, a_e, C_{em} = const) the *differential mass-transfer equation* takes the form

$$\frac{\partial \rho}{\partial \tau} = a_e\frac{C_e}{C_{em}}\frac{\partial^2 P_z}{\partial Z^2}. \tag{17}$$

Bearing in mind that specific mechanic energy flux transferred by diffusion during mass transfer is

$$q_e = C_{em} q_m, \quad \text{W/m}^2, \tag{18}$$

the differential energy transfer equation may be obtained for pressure shaping of flat articles from disperse mass.

For the one-dimensional problem considered

$$\frac{\partial e}{\partial \tau} = a_e\frac{\partial^2 e}{\partial Z^2}. \tag{19}$$

For zonal calculations (C_e = const)

$$\frac{\partial P_z}{\partial \tau} = a_e\frac{\partial^2 P_z}{\partial Z^2}. \tag{20}$$

Thus, a complicated process of pressure shaping of disperse mass involving changes in volume, mass concentration and energy in space and time (a nonlinear problem) may be approximately described by a system of relatively simple differential transfer equations (a linear problem)

$$\left.\begin{aligned}\frac{\partial \rho}{\partial \tau} &= a_e\frac{C_e}{C_{em}}\frac{\partial^2 P_z}{\partial Z^2}\\ \frac{\partial P_z}{\partial \tau} &= a_e\frac{\partial^2 P_z}{\partial Z^2}\end{aligned}\right. \tag{21}$$

which show clear relationship between local changes in mass density with the rate of pressure change at these points in the direction of a moulding process

$$\frac{\partial \rho}{\partial \tau} = \frac{C_e}{C_e}\frac{\partial P_z}{\partial \tau}. \tag{22}$$

Experimental and analytical checks of validity of the mathematical model may be performed if

(a) initial and boundary conditions for the process are known; (b) the differential energy transfer equation at the prescribed initial and boundary conditions is solved; (c) there are experimental data on pressure history over the surface and at the system centre; (d) energophysical coefficients entering into solution of the energy transfer equation are given.

A comparison of experimental and predicted curves of tensometric diagram $P_z \neq f(\tau)$ makes it possible to judge how the model approaches the real process.

Initial conditions comprise physical and geometric parameters of the system in its initial state ($\tau = 0$): such as granulometric composition of powder; initial density ρ_o, kg/m^3; energophysical parameters of the system; its geometric dimensions, temperature conditions.

To investigate general relationships of energy and mass transfer use has been made of electrolytic titanic powder PTM-I and metallurgic magnesite powder.

Geometric dimensions of a thermodynamic mass-saturated system are 155 × 50 × 25 mm.

Temperature conditions: at the initial moment $\tau = 0$ of moulding process and at any τ constant temperature of 293°K is assumed. Energy and physical parameters of the system entering into differential transfer equations are determined using a semi-analytical method which will be presented below.

Boundary conditions of packing process for disperse materials that determine behaviour of specific energy or potential (specific pressure) flux over the system surface may be subdivided into three main groups.

1. The first kind boundary conditions define behaviour of a specific pressure over the pressed piece surface, for example

(a) $P_s = \xi_p \tau$, where $\xi_p = \partial P_s/\partial \tau$ is the rate of specific pressure growth over the surface, N/m^2 s; τ is the time of moulding, s.

At ξ_p = const specific pressure growth is linear.

(b) P_s = const. This first kind boundary condition implies instantaneous pressure rise over the pressed piece surface that may be provided when a moulding process is performed using the explosion method.

2. The second kind boundary conditions define behaviour of specific energy flux q_e per energy-receiving surface that shows an energy amount from a plunger per pressed piece unit surface area ($1 m^2$) per unit time ($1 s$); $J/m^2 s$, W/m^2. For instance,

(a) the condition $q_e = \text{const}$ implies constant energy flux to the pressed piece surface within the process under consideration;

(b) the condition $q_e = \xi_q . \tau$ at $\xi_q = \partial q_e/\partial \tau = \text{const}$ implies a linear energy flux change during moulding process.

3. The third kind of boundary conditions imply mass packing because of rational use of a conservative force field acting on particles within the whole disperse system volume.

Such boundary conditions hold primarily for packing by vibration, in centrifuges, etc. Besides, more complicated boundary conditions may hold which imply the moulding process to run at various combination of the first and second kind of boundary conditions. For example:

(a) condition $P_s = \xi_p . \tau$; $P_{s_e} = \text{const}$, $(P_{s_e} = \xi_p . \tau_1)$ implies the process to be conducted with linear initial pressure growth over the pressed piece surface for time interval of τ_1 followed by keeping the pressed piece at final pressure P_{s_e};

(b) condition $q_e = \text{const}$; $P_{s_e} = \text{const}$ implies a constant energy flux over a pressed piece surface with its subsequent keeping at constant final pressure P_{s_e} of moulding;

(c) condition $P_{s_i} = \text{const}$ holds in case of mass packing by pneumoramming. In this case the process consists of a great number of "steps" (i) moulding at $P_s = \text{const}$ ($i = 1, 2, 3 \ldots \infty$), etc.

The boundary conditions of processes may be analysed from the energy diagram that represents a graphic interpretation of power changes such as working power (M_w), press idling power (M_{idl}) and intake power (M_{in}) for a moulding cycle.

M_w is the instantaneous power consumed by a press during moulding, W;

M_{idl} is the instantaneous power consumed by a press when developing a working pressure without power supply for energy and mass transfer in the considered system, W;

M_{in} is the power consumed for moulding per unit time, W;

$$M_{in} = q_e . F, W. \tag{23}$$

F is the energy and mass-receiving surface area of a thermodynamic system saturated with mass and energy, m^2.

For powder packing use has been made of PSU-500 press. Energy diagrams of the processes recorded by means of oscillograph-700 are presented in Fig. 2(b).

Analysis of the diagrams shows that moulding has proceeded with a linear growth of specific energy flux $q_e = \xi_q . \tau$; $(\xi_q = \partial q_e/\partial \tau = \text{const})$, i.e. under the second kind of boundary conditions.

Bearing in mind that differential potential conductivity equation (20) is derived with the assumption of constant electrical conductivity λ_e and constant energy capacity C_e for the system under consideration ($a_e = \lambda_e/C_e$) that is not so in reality. The initial and boundary conditions will now be formulated for the process considered with regard for necessity of zonal calculations at $\lambda_e, C_e, a_e = \text{const}$.

INITIAL CONDITIONS

The analysis reveals that for symmetric bilateral moulding the curves of pressure distribution over the cross-section of a flat pressed piece may be expressed for the first approximation by the second-kind parabola as

$$P_z(z, 0) = f_1(Z) \cong P_{co} + \frac{z^2}{R^2} \Delta P_o. \tag{24}$$

Here $P_z(z, 0)$ is the pressure at any point z over the cross-section of a pressed piece at any moment of moulding taken as a reference time ($\tau = 0$); R is the calculated plate thickness, m; $P_o = P_{so} - P_{co}$ is the pressure difference between the surface (P_{so}) and the centre of the pressed piece (P_{co}) at the initial moment of the considered region.

Boundary conditions for a solution of differential equation (24) for zonal calculations may be formulated as

$$q_e = q_o + \xi_q . \tau. \tag{25}$$

A solution of differential potential conductivity equation (24) for the given initial and boundary conditions is of the form

$$\begin{aligned} P(z, \tau) = P_{co} + \Delta P_o \Bigg[\tfrac{1}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2 \pi^2} \cos \frac{n\pi z}{R} \\ \times \exp(-n^2 \pi^2 Fo) \Bigg] \\ + \frac{q_{eo}}{\lambda_e} \Bigg[R . Fo - \frac{R^2 - 3z^2}{6R} + 2R \sum_{n=1}^{\infty} (-1)^{n+1} \\ \times \cos \frac{n\pi z}{R} \times \exp(-n^2 \pi^2 Fo) \Bigg] \\ + \frac{\xi_q . R^3}{\lambda_e a_e} \Bigg[\tfrac{1}{2} Fo^2 . Fo . \frac{z^2}{2R^2} - \tfrac{1}{6} Fo + \frac{z^4}{24R^4} \\ - \frac{z^2}{12R^2} + \frac{z}{360} - 2 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^4 \pi^4} \\ \times \cos \frac{n\pi z}{R} \times \exp(-n^2 \pi^2 Fo) \Bigg]. \end{aligned} \tag{26}$$

To compare experimental data with the predicted values by formula (26) $P_z = f(z, \tau)$, tensometric diagrams of a moulding process were taken $P_s = \varphi(\tau)$; $P_c = \psi(\tau)$.

Here P_s and P_c were recorded on oscillograms as averaged values with respect to mass and energy-receiving surfaces "external" and "central" (see Fig. 2).

Superposition of energy and tensometry diagrams makes it possible to evaluate coefficients entering into equation (4).

Mean values of specific energy capacity of the process are found from the balance equation

$$q_{em} . F . \tau = (C_e)_o^{P_m} . P_m . V \tag{27}$$

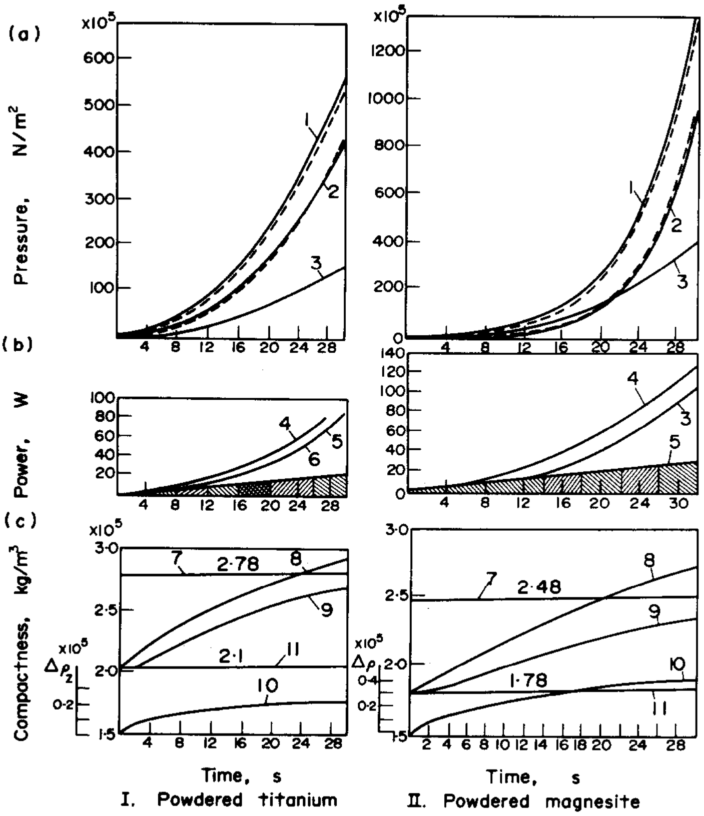

FIG. 2. Tensometry, energy and mass concentration diagrams of a packing process of a disperse plate (a) *Tensometry diagram* $P_z = \varphi(\tau)$; 1, mean pressure P_s over mass- and energy-surface of a disperse plate, N/m^2; 2, mean pressure P_c in a plane through the symmetry axis in the plate centre (at distance B from the surface, N/m^2); 3, pressure difference $\Delta P = P_s - P_c$, N/m^2. (b) *Energy diagram* $M = f(\tau)$; 4, working power, M_w, W; 5, idling power of power unit (a press), M_{idl}, W; 6, intake power, M_{in}, consumed by the thermodynamic system with mass transfer, W; (c) *Mass concentration diagram* $\rho = \psi(\tau)$; 7, mean initial density of a disperse plate, kg/m^3; 8, mean density over the mass-receiving surface, ρ_s, kg/m^3; 9, mean density in the axial plane of the plate, ρ_c, kg/m^3; 10, density difference between surface and centre of the plate, $\Delta\rho_z = \rho_s - \rho_c$, kg/m^3; 11, mean final density of intermediate products, ρ_e^m, kg/m^3.

hence

$$(C_e)_o^{P_m} = \frac{q_{em}\cdot F\tau}{P_m\cdot V} = \frac{\xi_q\cdot\tau^2}{2P_m\cdot R}, \tag{28}$$

$$(C)_{P_m^b}^{P_m^e} = \frac{(C_e)_o^{P_m^e}\cdot P_m^e - (C_e)_o^{P_m^b}\cdot P_m^b}{P_m^e} \tag{29}$$

Here q_{em} is the energy flux averaged within the considered interval ("zone") of the moulding process, J/m^2 s:

$$q_{em} = \frac{q_b + q_e}{2}.$$

P_m is the pressure averaged over the pressed piece cross-section, N/m^2; $P_m = P_s - \frac{2}{3}\Delta P$ with a parabolic pressure distribution; V is the volume of a thermodynamic system "saturated" with mass and energy; $C_o^{P_m}\cdot P_m$ is the compression energy per 1 m^3 of the considered system, J/m^3; $(C_e)_{P_m^b}^{P_m^e}$ is the specific energy capacity of the process averaged within the mean pressure range from P_m^b up to P_m^e in a pressed piece.

Mean values of energy conductivity (λ_e) are found with the assumption that within individual regions ($\Delta\tau$) moulding proceeds at q_{em} = const so

$$\Delta P_m = \frac{q_{em}\cdot R}{2(\lambda_e)_m}$$

i.e.

$$(\lambda_e)_m = \frac{q_{em}\cdot R}{2\Delta P_m}$$

where

$$\Delta P_m = \frac{\Delta P_e + \Delta P_b}{2}.$$

Mean values of potential conductivity for zonal intervals ($\Delta\tau$) are determined from predicted values of $(\lambda_e)_m$ and $(C_e)_m$:

$$(a_e)_m = \frac{(\lambda_e)_m}{(C_e)_m}, \quad \text{m}^2/\text{s}. \tag{30}$$

Table 1. Energophysical parameters of the thermodynamic system (matrix-electrolytic titanium powder PTM-1) shaped by pressure

Zones of calculation, No.	Time intervals	$P_m^e \times 10^3$	P_m^e	$(C_e)_o^{P_m^e}$	$(C_e)_{P_m^b}^{P_m^e}$	$(\lambda)_m$	$a_e \times 10^{-4}$	$\Delta\rho_m^b$	$\Delta\rho_m^e$	$(C_{em})_o^{\Delta\rho_m^e}$	$(C_{em})_{\Delta\rho_m^b}^{\Delta\rho_m^e}$
	(s)	(N/m^2)	(MN/m^2)	$\left(\frac{\text{J}}{\text{m}^3(\text{N/m}^2)}\right)$		$\left(\frac{\text{mW}}{\text{N/m}^2}\right)$	(m^2/s)	(kg/m^3)		$\left(\frac{\text{J}}{\text{m}^3(\text{kg/m}^3)}\right)$	
1	8	0	28	2·40	2·40	0·54	2·25	0	183	0·367	0·367
2	4	28	59	2·56	2·70	0·55	2·04	183	274	0·550	0·920
3	4	59	109	2·45	2·33	0·54	2·30	274	365	0·736	1·290
4	4	109	181	2·30	2·07	0·42	2·03	365	457	0·916	1·650
5	5	181	294	2·20	2·04	0·33	1·62	457	570	1·150	2·080
6	5	294	437	2·15	2·05	0·29	1·41	570	685	1·380	2·520

Note: $\xi_q = 71{\cdot}2\,\frac{\text{J}}{\text{m}^2\text{s}^2}$, $\rho_o = 2100\,\frac{\text{kg}}{\text{m}^3}$.

Table 2. Energophysical parameters of the thermodynamic system (matrix-metallurgic magnesite powder, fraction 0·5–2 mm) shaped by pressure

Zones of calculation, No.	Time intervals $\Delta\tau$	P_m^b	P_m^e	$(C_e)_o^{P_m^e}$	$(C_e)_{P_m^b}^{P_m^e}$	$(\lambda_e)_m$	$a_e \times 10^{-4}$	$\Delta\rho_m^b$	$\Delta\rho_m^e$	$(C_{em})_o^{\Delta\rho_m^e} \times 10^3$	$(C_{em})_{\Delta\rho_m^b}^{\Delta\rho_m^e} \times 10^3$
	(s)	(MN/m^2)	(MN/m^2)	$\left(\frac{\text{J}}{\text{m}^3(\text{N/m}^2)}\right)$		$\left(\frac{\text{mW}}{\text{m.N/m}^2}\right)$	$\left(\frac{\text{m}^2}{\text{s}}\right)$	(kg/m^3)		$\left(\frac{\text{J}}{\text{m}^3(\text{kg/m}^3)}\right)$	
1	9	0	20·0	1·231	1·231	0·1030	0·837	0	210	1·17	1·17
2	5	20·0	48·3	1·235	1·235	0·0860	0·700	210	326	1·82	3·05
3	4	48·3	95·6	1·030	0·821	0·0650	0·788	326	409	2·34	4·40
4	4	95·6	226·0	0·651	0·373	0·0440	1·189	409	513	2·87	4·92
5	4	226·0	501·2	0·410	0·212	0·0310	1·463	513	606	3·39	6·18
6	4	501·2	997·0	0·278	0·145	0·0250	1·745	606	699	3·91	7·32

Note: $\xi_q = 188{\cdot}3\ \text{J/m}^2\text{s}^2$, $\rho_o = 1780\,\text{kg/m}^3$.

Since the present energodynamic model of moulding implies clear interrelation between local changes in density and pressures

$$\frac{\partial \rho}{\partial \tau} = \frac{C_e}{C_{em}} \frac{\partial P_z}{\partial Z}$$

it seems useful to determine simultaneously specific energy capacity C_e and mean values C_{em} of specific energy capacity with mass transfer for individual zones of calculation. These values may be found from the balance equation

$$q_{em} . F . \tau = (C_{em})_o^{\Delta\rho_m} . \Delta\rho_m . V \qquad (31)$$

where $\Delta\rho_m$ is the density increment of the volume-averaged system for the considered period of moulding, kg/m^3.

Assuming for the first approximation a linear increase of the density of the system with linear movements of a movable plunger, we may obtain

$$\Delta\rho_m = \rho_o \frac{F}{V} \Delta l = \rho_o \frac{W_e . \tau}{R} \qquad (32)$$

hence

$$(C_{em})_o^{\Delta\rho_m} = \frac{\xi_q . \tau}{2\rho_o W_e} \qquad (33)$$

$$(C_{em})_{\Delta\rho_b}^{\Delta\rho_e} = \frac{C_o^{\Delta\rho_e} . \Delta\rho_e - C_o^{\Delta\rho_b} . \Delta\rho_b}{\Delta\rho_e - \Delta\rho_b}. \qquad (34)$$

Here W_e is the velocity of the plunger (m/s).

Predicted values of energophysical coefficients for the thermodynamic system under consideration are furnished in Tables 1 and 2.

Pressure distribution curves $P_s = \varphi(\tau)$ and $P_c = \psi(\tau)$ predicted from solution of equation (26) are presented in the tensometric diagrams (Fig. 2a). The same figure presents predicted densities of the pressed pieces over their surfaces and at the centre (at bilateral moulding) as $\rho_s = \varphi(\tau)$; $\rho_c = f(\tau)$.

Analysis of the curves in Fig. 2 shows that experimental and predicted curves $P = f(\tau)$ over the surface and in the centre of a pressed piece practically coincide. This proves the validity of the proposed energodynamic model of moulding.

CONCLUSION

A phenomenological model of energy and mass transfer is proposed for pressure shaping of disperse materials. The proposed model provides:

1. An approximate description of moulding;
2. Determination of optimum initial and boundary conditions of moulding processes (fractional composition, type of moisture, humidity, properties of press moulds and plungers, rate of specific pressure growth over the pressed piece surface, an effect of hold up time, behaviour of an energy flux, etc.) with minimum experimental data;
3. Prediction of behaviour of mass concentration at any point of a pressed piece at any moment of moulding under different initial and boundary conditions.

The proposed tensometry, energy and mass-concentration diagrams make it possible to present simultaneously changes of parameters of the present process in space and time.

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THEORIE PHENOMENOLOGIQUE DU TRANSFERT DE MASSE ET D'ENERGIE—MOULAGE SOUS PRESSION DES MATERIAUX DISPERSIFS

Résumé—L'article propose un modèle physico-mathématique de moulage qui permet de réduire un problème non-linéaire de transfert de masse et d'énergie avec déformation de volume en un problème linéaire de transfert de masse et d'énergie dans un système thermodynamique à volume constant.

PHÄNOMENOLOGISCHE THEORIE DES ENERGIE- UND STOFFTRANSPORTS BEIM FORMPRESSEN DISPERSER MATERIALIEN

Zusammenfassung—In dem Aufsatz wird ein physikalischmathematisches Modell für die Formgebung vorgeschlagen, welches das nichtlineare Problem des Wärme- und Stoffübergangs mit Volumenänderung auf ein lineares Problem des Wärme- und Stoffübergangs an ein thermodynamisches System konstanten Volumens reduziert.

ФЕНОМЕНОЛОГИЧЕСКАЯ ТЕОРИЯ ЭНЕРГО — И МАССОПЕРЕНОСА ПРИ ОБРАБОТКЕ ДИСПЕРСНЫХ МАТЕРИАЛОВ ДАВЛЕНИЕМ

Аннотация — В работе предложена физико-математическая модель процесса прессования, приводящая нелинейную задачу энерго- и массопереноса при деформации объема к линейной задаче переноса энергии и массы вещества в термодинамическую систему с постоянным объемом.